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SELF-ENERGY PECULIARITIES OF THE HOT GAUGE THEORY AFTER SYMMETRY BREAKING [1]

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Abstract

A tensor representation of the gluon propagator is found within covariant gauges for a non-Abelian theory after symmetry breaking due to $< A_0 >\neq 0$ and the exact equations which determine the dispersion laws of plasma excitations are explicitly obtained. In the high temperature region and fixing the Feynman gauge we solved these equations and found the damping of the plasma oscillations and the shifting of their frequency. The phase transition of a gauge symmetry restoration is estimated to be $\alpha_c(T) \approx 4/3$.

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1 Introduction

Today there is a hope that a non-Abelian gauge theory which acquires a new vacuum after the global gauge symmetry spontaneously breaking will be free of infrared divergencies [1,2] and able to give the reliable predictions for physics. A new vacuum is formed by a nonzero A_0-condensate which results from the infrared instability of a non-Abelian gauge theory and its appearence minimizes the thermodynamical potential beyond the trivial vacuum (the old scenario proposed many years ago [3]). This scenario was studied in connection with the deconfinement problem [4,5] where it was shown that the vacuum with $< A_0 >\neq 0$ breaks a $Z(N)$-symmetry and gauge theory loses their confining property in the high temperature region. Resently the scenario with $< A_0 >\neq 0$ was again studied in many papers (on the two-loop level [6-10] and with the higher order corrections taken into account [11,12], see also review [13] for details), but unfortunately up to now the possibility of A_0-condensation is not rigorous proved. The problem arises when the multi-loop corrections are taken into account and also this scenario is not confirmed by using the partition function found for the eigenvalues on the Wilson line [14]. Today this problem remains however it is not a reason to reject this scenario which opens the new possibilities for physics (the same opinion see in [13] and Ref.[15] is interesting as well). Of course, the question of gauge dependence of this phenomenon is also not completely clear. The A_0-quantity being a minimum of an effective action is gauge-dependent at the beginning but unfortunately the perturbative thermodynamical potential (namely its g^4-term and the higher terms as well) are also gauge-dependent [1] even for the case when gauge-covariance of perturbative calculations are checked by using the Nielsen identity [9].

The goal of this paper is to investigate the self-energy peculiarities of a non-Abelian gauge theory after symmetry breaking and to verify a selfconsistency of the scenario with $< A_0 >\neq 0$. This theory being more complicated requires a new formalism for clarifying its sence since at the beginning the self-energy tensor has the new structures which are found by using the gauge group indices (in accordance with a classical field vector). Our formalism is built in a covariant background gauge with an arbitrary α-paramiter and the exact dispersion equations are explicitly obtained. It is shown that there are the two singled out gauges (the Landau and Feynman ones) where these equations have a more simple form and we solved them in the high tem-

perature region by fixing the Feynman gauge. It is established that the plasma oscillations (both the transverse and longitudinal ones) have a finite damping everywhere even in the high temperature limit and the well-known plasmon frequency is shifted but differently for each vector field. Extending this scenario to the low temperature region we find the point T_c below which the A_0-condensate should be disappear to keep the squared plasma frequency positive. The estimation gives that the phase transition is localized in $\alpha_c(T) \approx 4/3$ and for the lower temperatures the gauge symmetry is restored.

2 Tensor structure of the gluon propagator

Our formalism is built by using the standard Green function technique at $T \neq 0$ in the background gauge with an arbitrary parameter α. To simplify our calculation we consider only the SU(2)-gauge group and the external classical field is chosen to be

$$\bar{A}^a_\mu = \delta_{\mu 4}\delta^{a3}\frac{\pi T}{g}x \tag{1}$$

Due to this choice there is a new vector in the theory and the self-energy tensor (which determines the standard gluon propagator) has the form

$$\Pi^{ab}_{\mu\nu} = \delta^{ab}\Pi_{\mu\nu} + K\bar{A}^a_\mu\bar{A}^b_\nu + R\,(\varepsilon^{afc}\bar{A}^f_\mu)(\varepsilon^{ctb}\bar{A}^t_\nu) \tag{2}$$

So the initial SU(2)- group is broken and the two tensors $\Pi^{||}_{\mu\nu}(\mathbf{p}_4, \mathbf{p})$ and $\Pi^{\perp}_{\mu\nu}(p_4, \mathbf{p})$ should be considered independly. Both these tensors are not transverse for any covariant gauges and, in a more general case, there are four scalar functions which determine them as follows

$$\Pi = \Pi_t \cdot A + \Pi_l \cdot B + \Pi_c \cdot C + \Pi_d \cdot D \tag{3}$$

where the four tensors are chosen to be [16]

$$A = I - B - D \;\;,\;\; B = \frac{V \circ V}{V^2} \;\;,\;\; C = \frac{Q \circ V + V \circ Q}{\sqrt{2}Q^2|\mathbf{q}|} \;\;,\;\; D = \frac{Q \circ Q}{Q^2} \tag{4}$$

which being in a covariant form are more convenient although they are only slightly different from the tensors used earlier in [17]. Here $V = Q^2U - (U \cdot$

$Q)Q$, and $U = (1, \mathbf{0})$ is the four-velocity of the thermal bath at rest. As usual $U^2 = 1$ and $V^2 = Q^2|\mathbf{q}|^2$. The algebra of these tensors is found as follows

$$\begin{aligned}
&A^2 = A \ , \ \ B^2 = B \ , \ \ D^2 = D \ , \ \ C^2 = \frac{1}{2}(B + D) \ , \\
&AB = AC = AD = BD = 0 \ , \ \ BC = CD = \frac{V \circ Q}{\sqrt{2}Q^2|\mathbf{q}|} \ , \\
&\frac{1}{2}\mathrm{Tr}A^2 = \mathrm{Tr}B^2 = \mathrm{Tr}C^2 = \mathrm{Tr}D^2 = 1 \qquad (5)
\end{aligned}$$

where I is the unit tensor and we use the Euclidean metric ($Q^2 = q_4^2 + |\mathbf{q}|^2$).

For the Feynman gauge (where $\alpha = 1$) there is a simplification and the polarization tensors for both sectors of the broken SU(2)-theory (instead of (3)) have a more simple structure [1]

$$\Pi = a \cdot (I - D) + \frac{b|\mathbf{q}|^2}{(U \cdot Q)^2} \cdot B + Z \cdot \frac{U \circ U}{(U \cdot Q)} \qquad (6)$$

where one can finds the following relations

$$\begin{aligned}
\Pi_t &= a, \qquad (7) \\
\Pi_l &= a + \frac{|\mathbf{q}|^2}{(U \cdot Q)^2}(b + Z\frac{(U \cdot Q)}{Q^2}); \ , \quad \frac{Z\sqrt{2}|\mathbf{q}|}{Q^2} = \Pi_c \ , \quad \frac{Z(U \cdot Q)}{Q^2} = \Pi_d
\end{aligned}$$

The representation (6) makes the Feynman gauge very convenient for the practical calculations and below namely this gauge will be used. But here within the exact algebra we continue to work with Eq.(3) and bearing in mind the standard definition of the inverse propagator

$$\mathcal{D}^{-1} = (Q^2 + \Pi_t) \cdot A + (Q^2 + \Pi_l) \cdot B + \Pi_c \cdot C + (\frac{Q^2}{\alpha} + \Pi_d) \cdot D \qquad (8)$$

the explicit form of the gluon propagator is found to be

$$\begin{aligned}
\mathcal{D} &= \frac{A}{Q^2 + \Pi_t} + \frac{1}{(Q^2 + \alpha\Pi_d)(Q^2 + \Pi_l) - \frac{\alpha}{2}\Pi_c^2}\Big\{(Q^2 + \alpha\Pi_d) \cdot B \\
&- \alpha\Pi_c \cdot C + \alpha(Q^2 + \Pi_l) \cdot D\Big\} \qquad (9)
\end{aligned}$$

that is valid for any α. In accordance with Eq.(9) there are two poles

$$Q^2 + \Pi_t = 0\,, \qquad (Q^2 + \alpha\Pi_d)(Q^2 + \Pi_l) - \frac{\alpha}{2}\Pi_c^2 = 0 \qquad (10)$$

which determine the modified dispersion equations for a gauge theory after the symmetry is spontaneously broken. For an arbitrary gauge the longitudinal oscillation spectrum is very complicated but there is one singled gauge $\alpha = 0$ where one finds Eqs.(10) in a rather simple form

$$Q^2 + \Pi_t = 0\,, \qquad Q^2 + \Pi_l = 0 \tag{11}$$

which is very close to the earlier known one. However, the Feynman gauge is also convenient since in this case the Π_l-function can be easily found from the self-energy diagram representation (through the Π_{44}-function). This equality is established to be

$$\Pi_l = \frac{Q^2}{|\mathbf{q}|^2}(\Pi_{44} - \frac{Z}{(Q \cdot U)}) + \frac{Z|\mathbf{q}|^2}{(Q \cdot U)Q^2} \tag{12}$$

and using Eq.(7) one can transform the dispersion equation for the longitudinal oscillations to the form

$$(Q^2 + \frac{Z(Q \cdot U)}{Q^2}) \left\{ 1 + \frac{1}{|\mathbf{q}|^2}(\Pi_{44} - \frac{Z}{(Q \cdot U)}) \right\} + \frac{Z|\mathbf{q}|^2}{Q^2(Q \cdot U)} = 0 \tag{13}$$

which for $|\mathbf{q}|^2 = 0$ reproduces a rather simple equation

$$1 + \frac{1}{|\mathbf{q}|^2}(\Pi_{44} - \frac{Z}{(Q \cdot U)}) = 0 \tag{14}$$

that allows to find the energy gap in the spectrum of the longitudinal oscillations with the aid of the simple calculation.

3 The high temperature spectra of plasma oscillations

Here we calculate the high temperature spectra of the plasma oscillations in the Feynman gauge (where $\alpha = 1$) and discuss their properties. Since the broken SU(2)-theory has two different sectors (named as "longitudinal" and "transverse" in accordance with its gauge group indices) there are two different expressions for each scalar function in Eq.(6).

3.1 Transverse oscillations in the SU(2)-transverse sector

The SU(2)-transverse sector contains two gauge fields $V^{\pm}_{\mu} = (V^1_{\mu} \mp iV^2_{\mu})/\sqrt{2}$ (with their ghost ones) and after the gauge symmetry breaking a new vacuum redefines the p_4-dependence for all functions within this sector. Now all momenta are shifted ($\hat{p}_4 = p_4 + \mu$ where $\mu = \pi T x$) and the polarization tensor is not transverse and has a structure like (6).

In the one-loop approximation this tensor is found to be [1]

$$
\begin{aligned}
-\Pi^{\perp}_{\mu\nu}(\hat{p}_4, \mathbf{p}) &= \frac{1}{\beta}\sum_{k_4}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\Big\{\frac{g^2}{k^2(k+\hat{p})^2}[\delta_{\mu\nu}(2k^2 + 5\hat{p}^2 + 2k\hat{p}) \\
&+ 8k_\mu k_\nu - 2\hat{p}_\mu\hat{p}_\nu + 4(k_\mu\hat{p}_\nu + k_\nu\hat{p}_\mu)] - 3\delta_{\mu\nu}g^2(\frac{1}{\hat{k}^2} + \frac{1}{k^2})\Big\}
\end{aligned}
\tag{15}
$$

where $\hat{p} = (\hat{p}_4, \mathbf{p})$ and $\hat{p}_4 = (p_4 + \mu)$ as it is mentioned above.

At first we calculate the $\Pi^{\perp}_{44}(\hat{p}_4, \mathbf{p})$-function within Eq.(15). The standard integrals are used and after all algebra being performed one finds that this function has the form

$$
\begin{aligned}
&\Pi^{\perp}_{44}(\hat{p}_4, \mathbf{p}) = \frac{g^2 N}{2\pi^2}\int\limits_0^\infty d|\mathbf{k}|\Big\{|\mathbf{k}|(\frac{n^+ + n^-}{2} + n) \\
&+\frac{1}{8|\mathbf{p}|}\Big[(\hat{p}_4^2 + 2\mathbf{p}^2 - 4\mathbf{k}^2)\big[(\frac{n^+ + n^-}{2} + n)\log(\hat{a}^+\hat{a}^-) - \frac{n^+ - n^-}{2}\log(\frac{\hat{a}^+}{\hat{a}^-})\big] \\
&+4|\mathbf{k}|(i\hat{p}_4)\big[(\frac{n^+ + n^-}{2} + n)\log(\frac{\hat{a}^+}{\hat{a}^-}) - \frac{n^+ - n^-}{2}\log(\hat{a}^+\hat{a}^-)\big]\Big]\Big\}
\end{aligned}
\tag{16}
$$

where $n = \{\exp[\beta|\mathbf{k}|] - 1\}^{-1}$ and all other abreviations are introduced to be

$$
\begin{aligned}
n^{\pm} &= \{\exp[\beta(|\mathbf{k}| \pm i\mu)] - 1\}^{-1} \\
\hat{a}^{\pm} &= \frac{(\mathbf{p}^2 + \hat{p}_4^2 - 2|\mathbf{k}||\mathbf{p}|) \pm 2i|\mathbf{k}|\hat{p}_4}{(\mathbf{p}^2 + \hat{p}_4^2 + 2|\mathbf{k}||\mathbf{p}|) \pm 2i|\mathbf{k}|\hat{p}_4}
\end{aligned}
\tag{17}
$$

In the leading order of T (for the high temperature region) the integral in Eq.(16) is calculated completely and the simple expression arises

$$
\Pi^{\perp}_{44}(\hat{p}_4, |\mathbf{p}|) = \frac{g^2 T^2 N}{\pi^2}[I_1(\frac{x}{2}) + I_1(0)]\Big\{1 - \frac{\xi}{2}\log\frac{\xi + 1}{\xi - 1}\Big\}
$$

$$+ \quad i\frac{g^2T^3N}{4\pi^2|\mathbf{p}|}I_2(\frac{x}{2})\log\frac{\xi+1}{\xi-1} \tag{18}$$

where $\xi = i\hat{p}_4/|\mathbf{p}|$ and the integrals I_i being treated in the dimensionless variable $|\mathbf{k}|/T$ are found to be

$$\begin{aligned} I_1(\frac{x}{2}) &= \int_0^\infty zdz\frac{n^+ + n^-}{2} = \pi^2 B_2(\frac{x}{2}) \\ I_2(\frac{x}{2}) &= i\int_0^\infty z^2dz(n^+ - n^-) = \frac{(2\pi)^3}{3}B_3(\frac{x}{2}) \end{aligned} \tag{19}$$

Here $x = \mu/\pi T$ and $B_i(z)$ are the standard Bernoulli polynomials

$$B_2(z) = z^2 - |z| + 1/6 \ \ , \ \ B_3(z) = z^3 - 3\varepsilon(z)z^2/2 + z/2 \tag{20}$$

with $\varepsilon(z) = z/|z|$ and $\varepsilon(0) = 0$.

To calculate the $\Pi_t^{\perp}(\hat{p}_4, \mathbf{p})$-function we use the convenient formula

$$\Pi_t = \frac{1}{2}\left\{\sum_i \Pi_{ii} - \frac{p_4^2}{\mathbf{p}^2}(\Pi_{44} - \frac{Z}{p_4})\right\} \tag{21}$$

which holds in the Feynman gauge in accordance with Eq.(6). We again return to Eq.(15) and use the formulas obtained above to perform the necessary algebra. The result has the form

$$\begin{aligned} \Pi_t^{\perp}(\hat{p}_4, |\mathbf{p}|) = \frac{g^2N}{4\pi^2}\int_0^\infty d|\mathbf{k}|\Bigg\{|\mathbf{k}|\Big[(1 - \frac{\hat{p}_4^2}{\mathbf{p}^2})(\frac{n^+ + n^-}{2} + n) + \frac{i|\mathbf{k}|\hat{p}_4}{\mathbf{p}^2}(n^+ - n^-)\Big] \\ + \frac{p^2}{8|\mathbf{p}|^3}\Bigg[(3\mathbf{p}^2 - \hat{p}_4^2 + 4\mathbf{k}^2)\Big[(\frac{n^+ + n^-}{2} + n)\log(\hat{a}^+\hat{a}^-) - \frac{n^+ - n^-}{2}\log(\frac{\hat{a}^+}{\hat{a}^-})\Big] \\ - 4|\mathbf{k}|(i\hat{p}_4)\Big[(\frac{n^+ + n^-}{2} + n)\log(\frac{\hat{a}^+}{\hat{a}^-}) - \frac{n^+ - n^-}{2}\log(\hat{a}^+\hat{a}^-)\Big]\Bigg]\Bigg\} \end{aligned} \tag{22}$$

where $p^2 = \mathbf{p}^2 + \hat{p}_4^2$ and we take into account that

$$\frac{Z^{\perp}}{\hat{p}_4} = i\frac{g^2T^3N}{2\pi^2(i\hat{p}_4)}I_2(\frac{x}{2}) \tag{23}$$

In the high temperature region Eq.(22) is simplified to be

$$\Pi_t^{\perp}(\hat{p}_4, |\mathbf{p}|) = \frac{g^2T^2N}{2\pi^2}(\xi^2-1)[I_1(\frac{x}{2})+I_1(0)]\left\{\frac{\xi^2}{\xi^2-1}-\frac{\xi}{2}\log\frac{\xi+1}{\xi-1}\right\} + i\frac{g^2T^3N}{8\pi^2|\mathbf{p}|}(\xi^2-1)I_2(\frac{x}{2})\left\{\log\frac{\xi+1}{\xi-1}-\frac{2\xi}{\xi^2-1}\right\} \tag{24}$$

where $\xi = i\hat{p}_4/|\mathbf{p}|$. Now the dispersion equation ($\hat{p}_4^2 + \mathbf{p}^2 + \Pi_t^{\perp} = 0$) for transverse plasma oscillations in the high temperature region has the form

$$(i\hat{p}_4)^2 = \frac{g^2T^2N}{2\pi^2}[I_1(\frac{x}{2})+I_1(0)]\,\xi^2\left\{\frac{\xi^2}{\xi^2-1}-\frac{\xi}{2}\log\frac{\xi+1}{\xi-1}\right\} + i\frac{g^2T^3N}{8\pi^2|\mathbf{p}|}I_2(\frac{x}{2})\,\xi^2\left\{\log\frac{\xi+1}{\xi-1}-\frac{2\xi}{\xi^2-1}\right\} \tag{25}$$

Eq.(25) has real and imaginary parts and determines both the spectrum of plasma oscillations and their damping. It is a rather complicated equation and we solve it only in the long-wave length limit for $\xi \to \infty$ where Eq.(25) is reduced to be

$$(i\hat{p}_4)^2 = \frac{g^2T^2N}{3\pi^2}[I_1(\frac{x}{2})+I_1(0)] - i\frac{g^2T^3N}{6\pi^2(i\hat{p}_4)}I_2(\frac{x}{2}) \tag{26}$$

Solving Eq.(26) we consider that $ip_4 = \omega = \Delta + i\Gamma$ and find two simple equations

$$\Delta^2 - (\Gamma+\mu)^2 = \frac{K(\Gamma+\mu)}{\Delta^2+(\Gamma+\mu)^2} + \Lambda^2 \ , \ 2(\Gamma+\mu) = \frac{K}{\Delta^2+(\Gamma+\mu)^2} \tag{27}$$

where the new abreviations are:

$$\Lambda^2 = \frac{g^2T^2N}{3\pi^2}[I_1(\frac{x}{2})+I_1(0)] \ , \ K = -\frac{g^2T^3N}{6\pi^2}I_2(\frac{x}{2}) \tag{28}$$

For the trivial vacuum (where $x = 0$) $\Lambda^2 = \omega_{pl}^2 = g^2T^2N/9$ and $K = 0$. Eq.(27) can be simplified and solved approximately for small g^2 where $4(\Gamma^2 + \mu) << \Lambda^2$. This solution has the form

$$\Delta^2 = 3(\Gamma+\mu)^2 + \Lambda^2 \ , \ 2(\Gamma+\mu) = K/\Lambda^2 \tag{29}$$

and for the physical vacuum where $\mu = \pi T x$ and $x = g^2/4\pi^2$ [1] one finds (if $g^2 << 1$) that

$$(\Gamma + \mu) \approx -\frac{g^2 T}{8\pi} \quad , \quad \omega^2 \approx 3(\frac{g^2 T}{8\pi})^2 + \frac{g^2 T^2 N}{9}(1 - \frac{3g^2}{8\pi^2}) \tag{30}$$

Studying Eq.(30) one can see that there is a point T_c where $\omega^2 = 0$. At this point the phase transition occurs and for all $T \leq T_c$ the gauge symmetry is restored. To etimate T_c we solve the equation $\omega^2 = 0$ and find that $\alpha_c(T) = g_c^2(T)/4\pi^2 \approx 4/3$ for the SU(2)-group. This is our main result for this setion but we would also like to stress that the same spectrum as (30) arises from Eq.(14) which determines the long-wave length limit for the longitudinal plasma oscillations. Indeed, using Eqs. (18) and (23) one finds that

$$\begin{aligned}(i\hat{p}_4)^2 &= \xi^2 \left\{ i\frac{g^2 T^3 N}{2\pi^2} I_2(\frac{x}{2}) \, [\frac{1}{(i\hat{p}_4)} - \frac{1}{2|\mathbf{p}|}\log\frac{\xi+1}{\xi-1}] \right. \\ &- \left. \frac{g^2 T^2 N}{\pi^2}[I_1(\frac{x}{2}) + I_1(0)] \, [1 - \frac{\xi}{2}\log\frac{\xi+1}{\xi-1}] \right\}\end{aligned} \tag{31}$$

and then taking the limit $\xi \to \infty$ within Eq.(31) we easily establish that Eq.(25) is exactly reproduced. This means that at the point $|\mathbf{p}| = 0$ the spectra of transverse and longitudinal oscillations coincide and determine the spectrum of the unique excitation with the three degrees of freedom.

3.2 Transverse oscillations in the SU(2)-longitudinal sector

The londgitudinal sector of the SU(2)-theory contains one real gauge field V_μ^3 (with its ghost one) and a new vacuum keeps the usual p_4-dependence (the own momenta are not shifted). Nevertheless the $\Pi^{||}_{\mu\nu}(p_4.\mathbf{p})$-tensor is not transverse the same as in the previous case and has a structure like (6).

In the one-loop approximation this tensor is found to be [1]

$$\begin{aligned}-\Pi^{||}_{\mu\nu}(p_4, \mathbf{p}) &= \frac{1}{\beta}\sum_{k_4}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\left\{\frac{g^2}{\hat{k}^2(\hat{k}+p)^2}[\delta_{\mu\nu}(2\hat{k}^2 + 5p^2 + 2\hat{k}p)\right. \\ &+ \left. 8\hat{k}_\mu\hat{k}_\nu - 2p_\mu p_\nu + 4(\hat{k}_\mu p_\nu + \hat{k}_\nu p_\mu)] - \frac{6\delta_{\mu\nu}g^2}{\hat{k}^2}\right\}\end{aligned} \tag{32}$$

where $\hat{k} = (\hat{k}_4, \mathbf{k})$ and $\hat{k}_4 = (k_4 + \mu)$. Here we calculate its scalar functions in accordance with Eq.(6) but below many details of these calculations will be omitted since they can be found in the previous section.

The result of this calculations is:

$$\Pi_t^{||}(p_4, |\mathbf{p}|) = \frac{g^2 N}{2\pi^2} \int\limits_0^\infty d|\mathbf{k}| \Big\{ |\mathbf{k}| \big[(1 - \frac{p_4^2}{\mathbf{p}^2}) \frac{n^+ + n^-}{2} + \frac{i|\mathbf{k}| p_4}{\mathbf{p}^2} (n^+ - n^-) \big]$$
$$+ \frac{p^2}{8|\mathbf{p}|^3} \Big[(3\mathbf{p}^2 - p_4^2 + 4\mathbf{k}^2) \big[\frac{n^+ + n^-}{2} \log(a^+ a^-) - \frac{n^+ - n^-}{2} \log(\frac{a^+}{a^-}) \big]$$
$$-4|\mathbf{k}|(ip_4) \big[\frac{n^+ + n^-}{2} \log(\frac{a^+}{a^-}) - \frac{n^+ - n^-}{2} \log(a^+ a^-) \big] \Big] \Big\} \tag{33}$$

where all abreviations are the same as previously. Here $a^\pm$ repeats Eq.(17) where $\hat{p}_4$ is replaced by p_4. In the high temperature region (if $p_4 \neq 0$ and $\xi = ip_4/|\mathbf{p}|$) one finds that Eq.(33) is simplified to be

$$\begin{aligned} \Pi_t^{||}(p_4, |\mathbf{p}|) &= \frac{g^2 T^2 N}{\pi^2} (\xi^2 - 1) I_1(\frac{x}{2}) \left\{ \frac{\xi^2}{\xi^2 - 1} - \frac{\xi}{2} \log \frac{\xi + 1}{\xi - 1} \right\} \\ &+ i \frac{g^2 T^3 N}{4\pi^2 |\mathbf{p}|} (\xi^2 - 1) I_2(\frac{x}{2}) \left\{ \log \frac{\xi + 1}{\xi - 1} - \frac{2\xi}{\xi^2 - 1} \right\} \end{aligned} \tag{34}$$

and the high temperature dispersion equation for the transverse plasma oscillations has the form

$$\begin{aligned} (ip_4)^2 &= \frac{g^2 T^2 N}{\pi^2} \, I_1(\frac{x}{2}) \, \xi^2 \left\{ \frac{\xi^2}{\xi^2 - 1} - \frac{\xi}{2} \log \frac{\xi + 1}{\xi - 1} \right\} \\ &+ i \frac{g^2 T^3 N}{4\pi^2 |\mathbf{p}|} I_2(\frac{x}{2}) \, \xi^2 \left\{ \log \frac{\xi + 1}{\xi - 1} - \frac{2\xi}{\xi^2 - 1} \right\} \end{aligned} \tag{35}$$

The same as previously Eq.(35) has real and imaginary parts and determines both the spectrum of plasma oscillations and their damping. It is a complicated equation as well and we solve it only in the long-wave length limit for $\xi \to \infty$ where Eq.(35) is reduced to be

$$(ip_4)^2 = \frac{2g^2 T^2 N}{3\pi^2} I_1(\frac{x}{2}) - i \frac{g^2 T^3 N}{3\pi^2 (ip_4)} I_2(\frac{x}{2}) \tag{36}$$

Here we consider that $ip_4 = \omega = \Delta + i\Gamma$ and find two simple equations

$$\Delta^2 - \Gamma^2 = \frac{K\Gamma}{\Delta^2 + \Gamma^2} + \Lambda^2 \ \ , \ \ 2\Gamma = \frac{K}{\Delta^2 + \Gamma^2} \tag{37}$$

where the new abreviations are:

$$\Lambda^2 = \frac{2g^2T^2N}{3\pi^2} I_1(\frac{x}{2}) \ \ , \ \ K = -\frac{g^2T^3N}{3\pi^2} I_2(\frac{x}{2}) \tag{38}$$

For the trivial vacuum (where $x = 0$) $\Lambda^2 = \omega_{pl}^2 = g^2T^2N/9$ and $K = 0$ the same as previously. Eq.(37) can be simplified and solved approximately for small g^2 where $4\Gamma^2 << \Lambda^2$. This solution has the form

$$\Delta^2 = 3\Gamma^2 + \Lambda^2 \ \ , \ \ 2\Gamma = K/\Lambda^2 \tag{39}$$

and for the physical vacuum where $x = g^2/4\pi^2$ [1] one finds (if $g^2 << 1$) that

$$\Gamma \approx -\frac{g^2T}{4\pi} \ \ , \ \ \omega^2 \approx 3(\frac{g^2T}{4\pi})^2 + \frac{g^2T^2N}{9}(1 - \frac{3g^2}{8\pi^2}) \tag{40}$$

In the end of this setion we again stress that the same spectrum as (40) arises from Eq.(14) which determines the long-wave length limit for the longitudinal plasma oscillations.

4 Conclusion

To summarize we have established a finite damping of plasma oscillations and the shifting of their frequency for the broken SU(2)-gauge theory when A_0-condensation takes place. This condensate occurs at the enough high temperature and disappears when the temperature falls below T_c: a temperature for which the plasma frequency of transverse oscillations in the SU(2)-transverse sector (see Eq.(30)) becomes equal to zero (here the running constant $\alpha_c(T) = g_c^2(T)/4\pi^2 \approx 4/3$). Below T_c the gauge symmetry restores and the theory again acquires the confining property. Our calculations are performed in the high temperature limit however we force to fix the Feynman gauge at the beginning to simplify many expressions and the algebraic transformations. Since we know [1] that in the used approximation the thermodynamical potential is gauge-dependent we do not exclude that

the one-loop results obtained for a self-energy are gauge-dependent as well although this dependence is not found explicitly. However we establish that there are two gauges (the Landau and Feynman ones) which are singled out for this task and the longitudinal spectrum of plasma oscillations essentially depends on this choice. Since this dependence arises on the agebraic level for the exact tensors there are no reasons to wait that the situation changes when the multi-loop corrections will be taken into account. Today the problem is to understand whether this dependence has a qualitative character or it leads only to a quantitative changes the physical results found within this theory. Of course, it is desirable to find a nonperturbative approach which allows to build the gauge-independent (or practically gauge-independent) thermodynamical potential. Within any perturbative calculations there is no chance to make the broken theory with $< A_0 > \neq 0$ to be gauge-independent although we can demonstrate that any of these approximations are gauge-covariant and self-consistent on the level of general identities (see [9] for the details).

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